A chosen form of a block matrix in a mathematical model of the induction squirrel-cage motors

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Abstract. In this article the usage of block matrices in modelling of the squirrel cage motors is presented. The relation between the structure of a block matrix and the self and mutual inductances coefficients is examined. The inversion method and determinant calculation of the block matrix of self and mutual inductances are shown. Exemplary numerical calculations are also presented to prove validity of the presented method of the matrix inversion.

Keywords: inverse matrix, determinant of a matrix, block matrix.

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1. Introduction

Commonly used mathematical models of electromechanical systems (including the squirrel cage induction motors) that is the systems with electrical and mechanical components coupled together, are often formulated by using the Kirchhoff's laws, Lagrange or Hamilton formalism. Their mathematical models are usually given by differential equations. Typically, these models are highly developed, for clarity they are often presented as a matrix. Some of these models can be written in the form of a symmetric block matrix, and the poliharmonic models of electromechanical converters – like squirrel cage induction motors – also belong to this group.

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2. The use of block matrices in the poliharmonic models of squirrel-cage motors

In [3, 4] there are presented in details the rules of deriving the poliharmonic models of squirrel-cage induction motors. In these works the graphical–analytical techniques, which significantly facilitate the process of formulating the mathematical models, are presented. These techniques allow the determination of the harmonic magnetic field in the air-gap of the motor, necessary to be taken into account in the mathematical model in order to ensure the satisfactory accuracy [7]. The problem of using the block matrices in formulating the mathematical models of squirrel-cage induction motor and drive system will be presented on the example of a system consisting of the engine and the drive system (composed from a number of inertial masses elastically connected). The mathematical model of a squirrel-cage induction motor (represented in the so-called biaxial coordinate system) and associated with the drive system of one or more degrees of freedom[1] [5], may be presented in the following system of differential equations

$$\frac{d}{dt}\begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} = \begin{bmatrix} M & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & J & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{1} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} u \\ T \\ \omega \end{bmatrix} - \begin{bmatrix} R + \omega_1 \frac{\partial}{\partial \theta_1} M & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & C & K \\ \mathbb{O} & \mathbb{O} & \mathbb{O} \end{bmatrix} \begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} \right\} \tag{1}$$

and an algebraic equation representing the electromagnetic torque produced by the motor:

$$T_e = i_s^T \frac{\partial}{\partial \theta_1} M_{sr} i_r \tag{2}$$

where M is a matrix of self and mutual inductances of the motor, J – a matrix of the mass moment of inertia, i – a vector of the motor currents, ω – a vector of the angular speed of rigid masses of the drive system, θ – a vector of the angular displacement of rigid masses, u – a stator supply voltages, T – a vector of the torques acting on rigid masses of the drive system, R – a matrix of the motor winding resistances, C – a matrix of the dumping coefficients, K – a matrix of the stiffness coefficients of the drive system shafts, ω_1 – an angular speed of rotor of the motor, θ_1 – an angular displacement of the rotor, T_e – an electromagnetic torque generated by the motor, i_s – a stator current vector, i_r – a rotor current vector, M_{sr} – a matrix of the stator-rotor mutual inductances. Model presented in Eq. (1) requires the block matrix inversion. Since the block matrix (1) is diagonal, its inverse consists of inverses of M, J and $\mathbb{1}$. Since J is also diagonal for the complex mechanical systems with lumped inertial masses connected by the elastic components (forming an open, serial kinematic chain), the only problematic part is M^{-1}. It is because M has a complex structure and contains the terms dependent on design parameters of the squirrel-cage induction motor such as: the number of pole pairs, the number of bars of the rotor cage, the number of spatial harmonics of the magnetic field in the air-gap. In the next chapter we will describe the relationship between the number of the magnetic field harmonics taken into account (also called spatial field harmonics) in the air-gap and the basic

[1] Drive system can be represented by a set of rigid inertial masses connected by the elastic shafts.

parameters such as: the number of pole pairs and the number of bars of the cage rotor.

3. The structure of the inductance matrix *M*

The structure of the inductance coefficients matrix M depends on the selected number of spatial harmonics of the magnetic field in the air-gap. These harmonics are generated by the symmetrical stator winding and rotor (cage) induction motor, which can be assigned to the so-called diagram of decomposition of the machines into the elementary machines [4] being a graphical representation of the interactions between the harmonics. This facilitates finding the generated asynchronous and pulsating (synchronous) electromagnetic torques – and hence the choice of harmonics included in the model. The graphical interpretation of the diagram of decomposition of a machine into elementary machines is shown in Fig. 1.

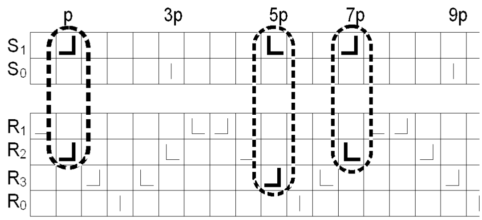

Fig. 1. Exemplary diagram of decomposition of the machine into the elementary machines for 2-pole pairs stator windings and 7 bar cage rotor

In the first two rows, denoted by S_1 and S_0, there are sequences of the generated magnetic field harmonics of the stator, which can be assigned to a sequence of the so-called monoharmonic elementary stators – a two phases or single phase (generating only one harmonic). Directions of rotation of the circular magnetic fields are taken into account by the orientation of axis of the coordinate system, direction of clock wise rotation mathematically positive (for example the coordinate system) is conventionally denoted by symbol – $\rfloor$, direction mathematically negative (opposite) – by symbol – $\lfloor$, while the component of zero magnetic field (magnetic field pulsed) uses symbol – $|$. In a similar manner indicated in Fig. 1 the sequences of harmonics generated by the rotor winding (squirrel cage) are marked in the rows from R_0 to R_3. The dashed lines in Fig. 1 indicate the harmonics of the stator and the rotor (single elementary machine) generating the asynchronous torques. So we have harmonics of orders p (equal to the number of pole pairs of the motor) – generating useful asynchronous torque, and harmonics (elementary machines) of higher orders – $5p$, $7p$ – generating asynchronous parasitic torques. For the elementary machines it can be assigned a matrix of coefficients representing the self-inductance and the matrices of coefficients representing the stator-rotor mutual inductance which have not a very complicated structure. These matrices are 4-element square matrices, and only in the case of the self-inductance matrix they are symmetrical. Individual form of the stator

– rotor mutual inductance matrix for the elementary machines can be determined by taking into account the rotation directions of magnetic fields and they were collected in tabular form in [3]. Generally, the inductance matrix for the poliharmonic mathematical models of squirrel cage motors will have the following block form

$$M = \begin{bmatrix} A & B & D & F & \cdots \\ B^T & C & \mathbb{O} & \mathbb{O} & \cdots \\ D^T & \mathbb{O} & E & \mathbb{O} & \cdots \\ F^T & \mathbb{O} & \mathbb{O} & G & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \tag{3}$$

where A, C, E, G are the diagonal matrices, B, D, F – the square 4-element matrices.

4. Searching for the inverted form of 9-element block matrix of type (3)

It is known that the inverse of the block matrix M in (3) is the adjugate matrix (which is symmetric and non-singular) multiplied by the reciprocal of its determinant, namely

$$M^{-1} = \frac{\operatorname{adj} M}{\det M}. \tag{4}$$

We will show how to determine both $\operatorname{adj} M$ and $\det M$.

4.1. The form of adjugate matrix

Multiplying the equality (4) by M, we obtain

$$M^{-1} M = \frac{\operatorname{adj} M}{\det M} M = \mathbb{1} \tag{5}$$

and it leads to

$$(\operatorname{adj} M)M = (\det M)\mathbb{1} \tag{6}$$

In the sequel, we will consider matrix of the form (3) of order 3, that is

$$M = \begin{bmatrix} A & B & D \\ B^T & C & \mathbb{O} \\ D^T & \mathbb{O} & E \end{bmatrix}. \tag{7}$$

Matrix (7) has the form of the matrix of inductance coefficients of the electromechanical transducer mathematical model, described in biaxial coordinate systems related to the stator and the rotor [4]. Therefore, we assume that the components A, C and E are diagonal matrices with the same elements on the main diagonal (due to the specific construction of mathematical model of the electromechanical transducer). Matrices B

and D are the full matrices. All the block matrices are square of order 2×2. Whence by (6) and (7) we get

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} A & B & D \\ B^T & C & \mathbb{O} \\ D^T & \mathbb{O} & E \end{bmatrix} = (\det M)\mathbb{1}, \tag{8}$$

where A_{ij} are proper blocks in $\operatorname{adj} M$ (each of order 2×2). The system (8) of matrix equations must be resolved for its unknown elements of matrix A_{ij}, which brings to solution of the following three systems of equations

$$\begin{cases} A_{11}A + A_{12}B^T + A_{13}D^T = (\det M)\mathbb{1} \\ A_{11}B + A_{12}C = \mathbb{O} \\ A_{11}D + A_{13}E = \mathbb{O} \end{cases} \tag{9}$$

$$\begin{cases} A_{21}A + A_{22}B^T + A_{23}D^T = \mathbb{O} \\ A_{21}B + A_{22}C = (\det M)\mathbb{1} \\ A_{21}D + A_{23}E = \mathbb{O} \end{cases} \tag{10}$$

and

$$\begin{cases} A_{31}A + A_{32}B^T + A_{33}D^T = \mathbb{O} \\ A_{31}B + A_{32}C = \mathbb{O} \\ A_{31}D + A_{33}E = (\det M)\mathbb{1} \end{cases} \tag{11}$$

The solutions of these system give the submatrices A_{ij} of the $\operatorname{adj} M$ in the form presented below

$$\begin{cases} A_{11} = \det M \cdot (A - BC^{-1}B^T - DE^{-1}D^T)^{-1} \\ A_{12} = -\det M \cdot (A - BC^{-1}B^T - DE^{-1}D^T)^{-1}BC^{-1} \\ A_{13} = -\det M \cdot (A - BC^{-1}B^T - DE^{-1}D^T)^{-1}DE^{-1} \end{cases} \tag{12}$$

$$\begin{cases} A_{21} = -\det M \cdot C^{-1}B^T(A - BC^{-1}B^T - DE^{-1}D^T)^{-1} \\ A_{22} = \det M \cdot (C - B^T(A - DE^{-1}D^T)^{-1}B)^{-1} \\ A_{23} = \det M \cdot C^{-1}B^T(A - BC^{-1}B^T - DE^{-1}D^T)^{-1}DE^{-1} \end{cases} \tag{13}$$

$$\begin{cases} A_{31} = -\det M \cdot E^{-1}D^T(A - BC^{-1}B^T - DE^{-1}D^T)^{-1} \\ A_{32} = \det M \cdot D^TE^{-1}(A - BC^{-1}B^T - DE^{-1}D^T)^{-1}BC^{-1} \\ A_{33} = \det M \cdot (E - D^T(A - BC^{-1}B^T)^{-1}D)^{-1} \end{cases} \tag{14}$$

Obviously, the above equalities hold under the assumption that all the inverse matrices presented in identities (12)–(14) exist. We can give the result in the more compact form. Namely, if we rewrite the first equality in (12) as

$$A_{11} = \det M \cdot a, \tag{15}$$

where $a = (A - BC^{-1}B^T - DE^{-1}D^T)^{-1}$ then (12)–(14) can be written as follows

$$\begin{cases} A_{11} = \det M \cdot a \\ A_{12} = -\det M \cdot aBC^{-1} \\ A_{13} = -\det M \cdot aDE^{-1} \end{cases} \tag{16}$$

$$\begin{cases} A_{21} = -\det M \cdot C^{-1}B^T a \\ A_{22} = \det M \cdot (C - B^T(A - DE^{-1}D^T)^{-1}B)^{-1} \\ A_{23} = \det M \cdot C^{-1}B^T aDE^{-1} \end{cases} \tag{17}$$

$$\begin{cases} A_{31} = -\det M \cdot E^{-1}D^T a \\ A_{32} = \det M \cdot D^T E^{-1}aBC^{-1} \\ A_{33} = \det M \cdot (E - D^T(A - BC^{-1}B^T)^{-1}D)^{-1} \end{cases} \tag{18}$$

Thus, the adjugate of M can be represented as

$$\operatorname{adj} M = \det M \left[\begin{array}{c|c|c} a & -aBC^{-1} & -aDE^{-1} \\ -C^{-1}B^T a & b & C^{-1}B^T aDE^{-1} \\ -E^{-1}D^T a & D^T E^{-1}aBC^{-1} & c \end{array}\right], \tag{19}$$

where $b = (C - B^T(A - DE^{-1}D^T)^{-1}B)^{-1}$, $c = (E - D^T(A - BC^{-1}B^T)^{-1}D)^{-1}$.

4.2. The form of determinant of the inductance block matrix

Now we search for the determinant of the matrix

$$M = \begin{bmatrix} A & B & D \\ B^T & C & \mathbb{O} \\ D^T & \mathbb{O} & E \end{bmatrix}. \tag{20}$$

Using the method described in section 4.1 or presented in [6], the determinant may be determined basing on the following equation

$$\begin{bmatrix} \mathbb{1} & \mathbb{O} & \mathbb{O} \\ X_1 & \mathbb{1} & \mathbb{O} \\ X_2 & X_3 & \mathbb{1} \end{bmatrix} \begin{bmatrix} A & B & D \\ B^T & C & \mathbb{O} \\ D^T & \mathbb{O} & E \end{bmatrix} = \begin{bmatrix} A & B & D \\ \mathbb{O} & Y_1 & Y_2 \\ \mathbb{O} & \mathbb{O} & Y_3 \end{bmatrix}, \tag{21}$$

In the paper [6] we discussed some generalisation of this equation, but here we are showing its application for determinant calculation of block matrices consisted of 9 elementary blocks. This equation must specify the form of the unknown matrices, first X_1, X_2 and X_3, then the matrices Y_1, Y_2 and Y_3. From (21) we get

$$X_1 A + \mathbb{1}B^T = \mathbb{O} \tag{22}$$

$$X_2 A + X_3 B^T + \mathbb{1}D^T = \mathbb{O} \tag{23}$$

$$X_2 B + X_3 C = \mathbb{O}. \tag{24}$$

From (22) we obtain

$$X_1 = -B^T A^{-1}. \tag{25}$$

Solving (23) and (24) yields

$$X_2 = D^T(BC^{-1}B^T - A)^{-1} \tag{26}$$

$$X_3 = -D^T(B^T - CB^{-1}A)^{-1} \tag{27}$$

whenever the respective inverse matrices on the right hand of (26) and (27) exist. The result substituted into (21) gives

$$\left[\begin{array}{c|c|c} \mathbb{1} & \mathbb{O} & \mathbb{O} \\ -B^T A^{-1} & \mathbb{1} & \mathbb{O} \\ D^T(BC^{-1}B^T - A)^{-1} & -D^T(B^T - CB^{-1}A)^{-1} & \mathbb{1} \end{array}\right] \begin{bmatrix} A & B & D \\ B^T & C & \mathbb{O} \\ D^T & \mathbb{O} & E \end{bmatrix} = \begin{bmatrix} A & B & D \\ \mathbb{O} & Y_1 & Y_2 \\ \mathbb{O} & \mathbb{O} & Y_3 \end{bmatrix}. \tag{28}$$

Whence we can eventually calculate Y_1, Y_2 and Y_3:

$$-B^T A^{-1} B + C = Y_1 \tag{29}$$

$$-B^T A^{-1} D = Y_2 \tag{30}$$

$$D^T(BC^{-1}B^T - A)^{-1} D + E = Y_3. \tag{31}$$

If we substitute (29)–(31) into (28), we get the following matrix equation

$$\begin{aligned} &\left[\begin{array}{c|c|c} \mathbb{1} & \mathbb{O} & \mathbb{O} \\ -B^T A^{-1} & \mathbb{1} & \mathbb{O} \\ D^T(BC^{-1}B^T - A)^{-1} & -D^T(B^T - CB^{-1}A)^{-1} & \mathbb{1} \end{array}\right] \begin{bmatrix} A & B & D \\ B^T & C & \mathbb{O} \\ D^T & \mathbb{O} & E \end{bmatrix} \\ &= \left[\begin{array}{c|c|c} A & B & D \\ \mathbb{O} & -B^T A^{-1} B + C & -B^T A^{-1} D \\ \mathbb{O} & \mathbb{O} & D^T(BC^{-1}B^T - A)^{-1} D + E \end{array}\right]. \end{aligned} \tag{32}$$

From properties of determinants (Cauchy equality) and from (32) we get

$$\begin{aligned} &\det \left[\begin{array}{c|c|c} \mathbb{1} & \mathbb{O} & \mathbb{O} \\ -B^T A^{-1} & \mathbb{1} & \mathbb{O} \\ D^T(BC^{-1}B^T - A)^{-1} & -D^T(B^T - CB^{-1}A)^{-1} & \mathbb{1} \end{array}\right] \det \begin{bmatrix} A & B & D \\ B^T & C & \mathbb{O} \\ D^T & \mathbb{O} & E \end{bmatrix} \\ &= \det \left[\begin{array}{c|c|c} A & B & D \\ \mathbb{O} & -B^T A^{-1} B + C & -B^T A^{-1} D \\ \mathbb{O} & \mathbb{O} & D^T(BC^{-1}B^T - A)^{-1} D + E \end{array}\right]. \end{aligned} \tag{33}$$

The first matrix is lower triangular with 1 on the main diagonal whence its determinant equals 1. The matrix on the right is upper triangular whence we finally get that

$$\det M = \det A \det(C - B^T A^{-1} B) \det(D^T(BC^{-1}B^T - A)^{-1} D + E). \tag{34}$$

We note that this formula is compatible with the ones obtained in paper [7].

4.3. Form of the inverted block matrix

Substituting (19) and (34) into (4) we get

$$M^{-1} = \left[\begin{array}{c|c|c} a & -aBC^{-1} & -aDE^{-1} \\ -C^{-1}B^T a & b & C^{-1}B^T aDE^{-1} \\ -E^{-1}D^T a & D^T E^{-1} aBC^{-1} & c \end{array}\right]^T, \tag{35}$$

where $a = (A - BC^{-1}B^T - DE^{-1}D^T)^{-1}$, $b = (C - B^T(A - DE^{-1}D^T)^{-1}B)^{-1}$, $c = (E - D^T(A - BC^{-1}B^T)^{-1}D)^{-1}$. It can be shown that between the submatrices of the inverted form of inductance matrix M the following relationships occur: $A_{21} = A_{12}^T$, $A_{31} = A_{13}^T$, $A_{32} = A_{23}^T$. But before the demonstration let us check whether the following relationship holds

$$a = a^T. \tag{36}$$

By using the properties of the matrix transpose operation with respect to the matrix inversion, we get

$$[(A - BC^{-1}B^T - DE^{-1}D^T)^{-1}]^T = [(A - BC^{-1}B^T - DE^{-1}D^T)^T]^{-1}, \tag{37}$$

then, since the transposition of sum of the matrices equals to the sum of transposed matrices we obtain

$$[(A - BC^{-1}B^T - DE^{-1}D^T)^{-1}]^T = (A^T - (BC^{-1}B^T)^T - (DE^{-1}D^T)^T)^{-1}. \tag{38}$$

At last, because the transposition of product of the matrices equals to the product of transposed matrices but in reverse order, we have

$$\begin{aligned} &[(A - BC^{-1}B^T - DE^{-1}D^T)^{-1}]^T \\ &\qquad = (A^T - (B^T)^T(C^{-1})^T B^T - (D^T)^T(E^{-1})^T D^T)^{-1}, \end{aligned} \tag{39}$$

By assumption, A, C and E, are diagonal, hence equal to their transpositions, and we obtain

$$(A - BC^{-1}B^T - DE^{-1}D^T)^{-1} = (A - BC^{-1}B^T - DE^{-1}D^T)^{-1}, \tag{40}$$

which proves the equality (36). Similarly we can prove that $A_{21} = A_{12}^T$, $A_{31} = A_{13}^T$, $A_{32} = A_{23}^T$. Finally we obtain

$$M^{-1} = \left[\begin{array}{c|c|c} a & -aBC^{-1} & -aDE^{-1} \\ (-aBC^{-1})^T & b & C^{-1}B^T aDE^{-1} \\ (-aDE^{-1})^T & (C^{-1}B^T aDE^{-1})^T & c \end{array}\right], \tag{41}$$

where $a = (A - BC^{-1}B^T - DE^{-1}D^T)^{-1}$, $b = (C - B^T(A - DE^{-1}D^T)^{-1}B)^{-1}$, $c = (E - D^T(A - BC^{-1}B^T)^{-1}D)^{-1}$. It is the right moment to remaind that the formula (41) holds whenever on the right hand of (41) all the respective inverse matrices exist.

5. An example of application

Suppose that the model of electromechanical converter is given by the matrix equations formulated for the four pole motor with the squirrel cage consisting of 28 cage bars

$$
\begin{aligned}
\frac{d}{dt}\begin{bmatrix} i_s^{\alpha\beta} \\ i_{r2}^{dq} \\ i_{r10}^{dq} \end{bmatrix} &= \left[\begin{array}{c|c|c} L_{\sigma s}^{\alpha\beta}+M_{ss}^{\alpha\beta} & M_{sr2}^{\alpha\beta dq} & M_{sr10}^{\alpha\beta dq} \\ (M_{sr2}^{\alpha\beta dq})^T & L_{\sigma r2}^{\alpha\beta}+M_{rr2}^{\alpha\beta} & \mathbb{O} \\ (M_{sr10}^{\alpha\beta dq})^T & \mathbb{O} & L_{\sigma r10}^{\alpha\beta}+M_{rr10}^{\alpha\beta} \end{array}\right]^{-1} \\
&\times\left\{\begin{bmatrix} u_s^{\alpha\beta} \\ 0 \\ 0 \end{bmatrix} - \left[\begin{array}{c|c|c} R_s^{\alpha\beta} & \omega_1\frac{\partial}{\partial\vartheta_1}M_{sr2}^{\alpha\beta dq} & \omega_1\frac{\partial}{\partial\vartheta_1}M_{sr10}^{\alpha\beta dq} \\ \omega_1\frac{\partial}{\partial\vartheta_1}(M_{sr2}^{\alpha\beta dq})^T & R_{r2}^{\alpha\beta} & \mathbb{O} \\ \omega_1\frac{\partial}{\partial\vartheta_1}(M_{sr10}^{\alpha\beta dq})^T & \mathbb{O} & R_{r10}^{\alpha\beta} \end{array}\right]\begin{bmatrix} i_s^{\alpha\beta} \\ i_{r2}^{dq} \\ i_{r10}^{dq} \end{bmatrix}\right\},
\end{aligned}
\tag{42}
$$

and the torque equation has the form

$$
T_e = (i_s^{\alpha\beta})^T\frac{\partial}{\partial\vartheta_1}\left[M_{sr2}^{\alpha\beta dq}\middle|M_{sr10}^{\alpha\beta dq}\right]\begin{bmatrix} i_{r2}^{dq} \\ i_{r10}^{dq} \end{bmatrix}, \tag{43}
$$

where R_{r2}^{dq}, R_{r10}^{dq}, $L_{\sigma r2}^{dq}$, $L_{\sigma r10}^{dq}$ are diagonal matrices of resistances and inductances related to the harmonics belonging to 2^{nd} and 10^{th} row of the diagram of decomposition, M_{rr2}^{dq}, M_{rr10}^{dq} are matrices of self-inductances related to the harmonics belonging to 2^{nd} and 10^{th} row of the diagram of decomposition, $M_{sr2}^{\alpha\beta dq}$, $M_{sr10}^{\alpha\beta dq}$ – matrices of self-inductances related to the harmonics belonging to 2^{nd} and 10^{th} row of the diagram of decomposition, i_{r2}^{dq} – vector of rotor currents related to the harmonics belonging to 2^{nd} row, i_{r10}^{dq} – vector of rotor currents related to the harmonics belonging to 10^{th} row.

In this model, in order to facilitate the analysis we introduce the same notation of the matrix block as in (1), that is

$$
\begin{aligned}
\frac{d}{dt}\begin{bmatrix} i_s^{\alpha\beta} \\ i_{r2}^{dq} \\ i_{r10}^{dq} \end{bmatrix} &= \begin{bmatrix} A & B & D \\ B^T & C & \mathbb{O} \\ D^T & \mathbb{O} & E \end{bmatrix}^{-1} \\
&\times\left\{\begin{bmatrix} u_s^{\alpha\beta} \\ 0 \\ 0 \end{bmatrix} - \left[\begin{array}{c|c|c} R_s^{\alpha\beta} & \omega_1\frac{\partial}{\partial\vartheta_1}B & \omega_1\frac{\partial}{\partial\vartheta_1}D \\ \omega_1\frac{\partial}{\partial\vartheta_1}B^T & R_{r2}^{\alpha\beta} & \mathbb{O} \\ \omega_1\frac{\partial}{\partial\vartheta_1}D^T & \mathbb{O} & R_{r10}^{\alpha\beta} \end{array}\right]\begin{bmatrix} i_s^{\alpha\beta} \\ i_{r2}^{dq} \\ i_{r10}^{dq} \end{bmatrix}\right\},
\end{aligned}
\tag{44}
$$

and for the torque equation

$$
T_e = (i_s^{\alpha\beta})^T\frac{\partial}{\partial\vartheta_1}\left[B\middle|D\right]\begin{bmatrix} i_{r2}^{dq} \\ i_{r10}^{dq} \end{bmatrix}. \tag{45}
$$

Block matrices appearing in equations (46) and (47) have the following form

$$
A=\begin{bmatrix} L_s & \mathbb{O} \\ \mathbb{O} & L_s \end{bmatrix},\ B=\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix},\ C=\begin{bmatrix} L_{r2} & \mathbb{O} \\ \mathbb{O} & L_{r2} \end{bmatrix},\ D=\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix},\ E=\begin{bmatrix} L_{r10} & \mathbb{O} \\ \mathbb{O} & L_{r10} \end{bmatrix}.
$$

The detailed form of elements of matrices B and D are as follows

$$
\begin{aligned}
b_{11} &= L_{sr2}\cos 2\vartheta_1 + L_{sr26}\cos 26\vartheta_1, & b_{12} &= -L_{sr2}\sin 2\vartheta_1 + L_{sr26}\sin 26\vartheta_1,\\
b_{21} &= L_{sr2}\sin 2\vartheta_1 + L_{sr26}\sin 26\vartheta_1, & b_{22} &= L_{sr2}\cos 2\vartheta_1 - L_{sr26}\cos 26\vartheta_1,\\
d_{11} &= -d_{22} = L_{sr10}\cos 10\vartheta_1, & d_{12} &= d_{21} = -L_{sr10}\sin 10\vartheta_1,
\end{aligned}
$$

where L_{sr2}, L_{sr10}, L_{sr26} are mutual inductance coefficients related to 2, 10 and 26 space harmonics, respectively.

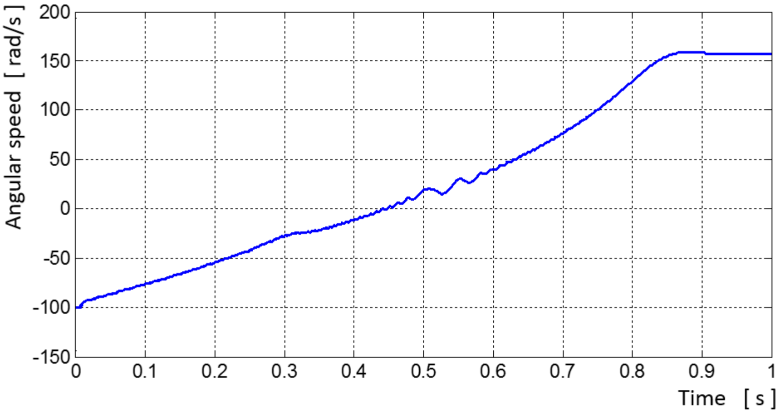

Fig. 2. Plot of the angular speed of rotor during the rotation direction reversal in dependance on time

Figure 2 shows the result obtained by numerical calculations (with the use of integration method “ode45” in Matlab/Simulink) showing the conversion of the angular speed of rotor of the motor during reversal (the reverse direction of the rotor rotation). It summarizes two runs obtained from the simulation model by applying the standard method for matrix inversion (“inv” method in Matlab) and the method which uses the block matrices during calculations. The results representing the differences in the absolute angular velocity at certain moments of time are shown in Fig. 3. During the simulation the following parameters were used $R_s^{\alpha\beta} = 2.47\Omega$, $R_{r2}^{dq} = 1.95e^{-5}\Omega$, $R_{r10}^{dq} = 1.84e^{-4}\Omega$, $L_s = 1.09H$, $L_{r2} = 8.79e^{-6}H$, $L_{r10} = 7.60e^{-6}H$, $L_{sr2} = 0.003H$, $L_{sr10} = 1.12e^{-4}H$, $L_{sr26} = 4.09e^{-6}H$ and mass moment of inertia $J = 0.05\ kgm^2$.

We note that the presented method is characterized by the significantly shorter times of computations than the standard methods. Proposed method has also the qualities predisposing it to the parallel computations.

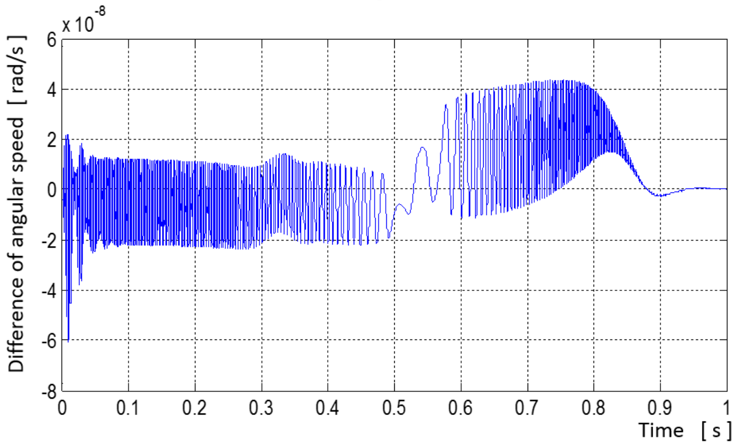

Fig. 3. Difference between the angular rotation speeds in dependance of time

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